Nagarjuna Degree College
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DCMT101

Reg. No. | | | | | | | | |

I Semester B.Sc. Degree Examination, May/June - 2022

MATHEMATICS

Algebra - I and Calculus - I

(NEP - CORE Scheme 2021-22 and Onwards)

Paper :I MATDSCT 1.1

Time : 2½ Hours **Maximum Marks : 60**

Instructions to Candidates:

Answer all questions.

I. Answer any **Six** questions. **(6×2=12)**

1. Find the value of k in order that the matrix

$$A = \begin{bmatrix} 6 & k & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix} \text{ is of rank 2.}$$

2. Find the eigen values of the matrix $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$.

3. Find the n^{th} derivative of $e^{2x}\cos 3x$.

4. If $u = x^2yz$ prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

5. Evaluate $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)$.

6. State Lagrange's mean value theorem.

7. Discuss the continuity of $f(x) = \begin{cases} 3x+1 & x>1 \\ 2x-1 & x \le 1 \end{cases}$ at $x = 1$.

8. Evaluate $\lim_{x\to 0} \frac{x-\sin x}{x^3}$.

[P.T.O.

II. Answer any **Three** questions. **(3×4=12)**

9. Find the rank of the matrix $A=\begin{bmatrix}1 & 1 & 1 & 2\\ 2 & 1 & -3 & -6\\ 3 & -3 & 1 & 2\end{bmatrix}$ by reducing it to normal form.

10. Find λ and μ such that the system of equations

$$\begin{aligned} x+3y+4z&=5\\ x+2y+z&=3\\ x+3y+\lambda z&=\mu \end{aligned}$$ has

i. no solution

ii. unique solution.

iii. many solutions.

11. Find the eigen values and the corresponding eigen vectors of the matrix.

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}.$$

12. State and prove Cayley - Hamilton theorem.

13. By using Cayley - Hamilton theorem. Find the adjoint of the matrix $\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}$.

III. Answer any **Three** questions. **(3×4=12)**

14. Discuss the continuity of $f(x)=\begin{cases} x^2-1 & for\ x<1\\ 1-\dfrac{1}{x} & for\ x>1\\ 0 & for\ x=1\end{cases}$ at $x=1$.

15. Examine the differentiability of $f(x)=\begin{cases} x^2, & x<3 \\ 6x-9, & x\geq 3 \end{cases}$ at $x=3$.

16. Find the n[th] derivative of $\frac{4x}{(x+1)^2(x-1)}$.

17. Prove that a function which is continuous in a closed interval attains its bounds in the interval.

18. If $y=\sin(m\sin^{-1}x)$, prove that $\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-(n^2-m^2)y_n=0$.

IV. Answer any **Three** questions. **(3×4=12)**

19. State and prove Rolle's theorem.

20. State and prove Cauchy's Mean value theorem.

21. Expand $\log(1+\sin x)$ upto the term containing x^4 using Maclaurin's series.

22. Evaluate $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^2 x}$.

23. Evaluate $\lim_{x\to 0}(1+\sin x)^{\cot x}$.

V. Answer any **Three** questions. **(3×4=12)**

24. If $u=f(r)$ where $r=\sqrt{x^2+y^2+z^2}$ show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f''(r)+\frac{2}{r}f'(r)$.

25. State the prove Euler's theorem on Homogeneous function.

26. If $u=x^2$, $v=y^2$ find $J=\frac{\partial(u,v)}{\partial(x,y)}$ and $J'=\frac{\partial(x,y)}{\partial(u,v)}$ also verify that $JJ'=1$.

27. Expand $e^x\cos y$ in a Taylor's series about the point at $(1,\pi/4)$ upto second degree terms.

28. Find the extreme value of the function $f(x,y)=x^3+y^3-3x-12y+2$.