I Semester B.C.A. Degree Examination, March/April - 2023

## COMPUTER APPLICATIONS

Discrete Mathematics
(CBCS Scheme)
Time: $\mathbf{3}$ Hours
Maximum Marks : 100

## Instructions to Candidates:

Answer all Sections.

## SECTION - A

I. Answer any TEN of the following.

1) Define power set with an example.
2) If $\mathrm{A}=\{2,3,5\} \mathrm{B}=\{4,5,6\} \mathrm{C}=\{1,2\}$ find $\mathrm{A} \times(\mathrm{B}-\mathrm{C})$.
3) Construct truth table for $-p V q$.
4) Define scalar matrix with an example.
5) State Caley-Hamilton theorem.
6) If $A=\left[\begin{array}{cc}2 & 1 \\ 4 & -2\end{array}\right] B=\left[\begin{array}{cc}-3 & 2 \\ 1 & 4\end{array}\right]$ then find $2 A-3 B$.
7) Prove that $\left(\log _{b}^{a}\right) \cdot\left(\log _{c}^{b}\right) \cdot\left(\log _{a}^{c}\right)=1$
8) If ${ }^{n} C_{8}={ }^{n} C_{9}$ then find ${ }^{n} C_{17}$.
9) Define Abelian group.
10) If $\vec{a}=2 \hat{i}+3 \hat{j}+4 \hat{k} \vec{b}=\hat{i}+2 \hat{j}+2 \hat{k}$ then find $|\vec{a}-\vec{b}|$.
11) Find the distance between the points $\mathrm{A}(-2,3)$ and $\mathrm{B}(-4,5)$.
12) Find the equation of straight line passing through $(2,5)$ and having slope 4 .
II. Answer any SIX of the following.
13) If $\mathrm{A}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}, \mathrm{B}=\{\mathrm{c}, \mathrm{d}, \mathrm{e}\} \mathrm{C}=\{\mathrm{c}, \mathrm{e}, \mathrm{f}, \mathrm{g}\}$ then verify $\mathrm{A} \cdot(\mathrm{B}-\mathrm{C})=(\mathrm{A} \cdot \mathrm{B})-(\mathrm{A} \times \mathrm{C})$.
14) If $f: R \rightarrow R$ defined by $f(x)=\mu x-3$ then prove that $f$ is invertible, also find inverse of $f$.
15) If $A=\left[\begin{array}{cc}1 & 2 \\ -3 & 1 \\ 5 & 4\end{array}\right] B=\left[\begin{array}{lrr}1 & 2 & -3 \\ -1 & 4 & 2\end{array}\right]$ then prove that $(A B)^{1}=B^{\prime} A^{\prime}$.
16) Solve the following system of equations using Cramer's rule

$$
\begin{aligned}
& 3 x+y+z=3 \\
& 2 x+2 y+5 z=-1 \\
& x-3 y-4 z=2
\end{aligned}
$$

17) Verify Caley-Hamilton theorem for the Matrix $A=\left[\begin{array}{cc}3 & -1 \\ 1 & 2\end{array}\right]$ also find $A^{-1}$.
18) Prove that $(P \rightarrow q) \leftrightarrow(\sim q \rightarrow \sim p)$ is a Tautology.
19) Prove that $p \wedge(q \vee r) \cong(p \wedge q) \vee(p \wedge r)$.
20) Write the converse, Inverse and contrapositive of " If two integers are equal then their squares are equal".

## SECTION - C

## III. Answer any SIX of the following.

21) If $\mathrm{a}^{3}+\mathrm{b}^{3}=\mathrm{ab}(8-3 \mathrm{a}-3 \mathrm{~b})$, then show that $\log \left(\frac{a+b}{2}\right)=\frac{1}{3}(\log \mathrm{a}+\log \mathrm{b})$.
22) In how many ways can the letters of the word "ASSASSINATION" be arranged so that all S's are not together.
23) A Examination question paper consists of 12 questions divided inte part $A$ and Part $B$ 8 consists of 7 questions and 5 questions respectively. A student is required to attempt 8 questions, selecting atleast 3 from each part. In how many ways can a student select the questions.
24) Prove that $\mathrm{G}=\{0,1.2,3,4,5\}$ is an abelian group under addition modulo 6 .
25) Show that the set of all fourth roots of unity form a group under multiplication.
26) If $\vec{a}=\hat{i}-\hat{j}+2 \hat{k} \quad \vec{b}=2 \hat{i}+3 \hat{j}-\hat{k}$ then find $(\vec{a}+2 \vec{b})(2 \vec{a}-\vec{b})$.
27) Show that the points $A(1,2,3) B(2,3,1)$ and $C(3,1,2)$ are the vertices of an equilateral triangle .
28) If the vectors $4 \hat{i}+11 \hat{j}+m \hat{k}, 7 \hat{i}+2 \hat{j}+6 \hat{k}$ and $\hat{i}+5 \hat{j}+4 \hat{k}$ are coplanar, then find ' $m$ '.

## SECTION - D

## IV. Answer any FOUR of the following.

29) Prove that the points $(4,-4),(8,2),(14,-2)$ and $(10,-8)$ are the vertices of square.
30) The three vertices of a parallelogram taken in order are (8,5), (-7, -5), (-5,5). Find the coordinates of the fourth vertex.
31) Find the equation of locus of point which moves such that it is equidistant from the points ( 1,2 ) and ( $2,-3$ ).
32) Derive the equation of the line whose $X$ - intercept is ' $a$ ' and $Y$-intercept is ' $b$ '.
33) If the line $2 x-5 y+1=0$ is perpendicular to $(p+1) x+(2 p+3) y+3=0$ then find $p$.
34) Find the equation of line passing through the point of intersection of $2 x+3 y-1=0$ and $3 x+4 y-6=0$ and parallel to the line $5 x-y=0$.
